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The 'art' of ratio:

Using a hands-on ratio task to explore student thinking


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The importance of ratio and proportional reasoning is regularly emphasised, however, the teaching and learning of these concepts continues to present challenges. In this article the authors explore a hands-on task, designed to encourage student visualisation of the concept of ratio. Teacher perceptions of this task were positive, indicating the potential of this task to impact on students' engagement and understanding of ratio.

Proportional reasoning is considered a milestone in students' mathematical development (Cramer & Post, 1993). However, proportional reasoning and ratio continue to be considered areas of mathematics that present considerable challenges for many students. Proportional reasoning is complex, requiring a thorough knowledge of ratio and proportion along with fractions, decimals, scale drawing and ratio (Siemon, Bleckly, & Neal, 2012); each of which rely on students having a solid conceptual understanding of multiplication and division. One of the reasons cited for students experiencing difficulties with ratio and proportion, is that many students intuitively apply additive strategies rather than using multiplicative thinking (Siemon et al., 2012). This issue of reliance on additive thinking is considered to be a significant challenge to teaching and learning proportional reasoning. For example, when faced with a ratio such as $2:5 = 6:?$, some students use additive thinking and may find the difference between 6 and 2, and then add this to five to incorrectly conclude that $2:5 = 6:9$. The development of proportional thinking is often described as a complex but gradual process moving from absolute to relativity (Lamon, 2005). However, despite this complexity, research suggests that with adequate support and scaffolding children as young as six years of age can acquire a basic understanding of proportionality (Carraher, 1996; Behr, Harel, Post, & Lesh, 1992). Early explorations of change, and working on concepts which anticipate ratio, can be useful in scaffolding students' conceptual development. For example, working with experiences such as sand and water play: "if it takes three cups of sand to make one sand pie, it will take nine cups to make three sand pies" (Fielding-Wells, Dole, & Makar, 2014, p. 47).

This paper presents an example of a hands-on, meaningful task designed to identify whether students are thinking additively or multiplicatively. Meaningful tasks can increase student motivation and also challenge them to think about the relevant mathematical concepts, connection to other aspects of mathematics and real-world relevance (National Council of Teachers of Mathematics, 1991; Schoenfeld & Kilpatrick, 2008). Meaningful and authentic tasks, particularly those that allow collaboration and fun, enhance student engagement by providing opportunities for students to develop their own meaning and have some form of ownership of the task which leads to sustained improvement in learning outcomes (Attard, 2011). In addition, the task is also designed to support students in visualising their thinking and self-correcting where possible. Visual representations support students to experiment with their thinking and promotes self-correction, thus supporting students in being autonomous learners, encouraging them to find solutions and reflect on their processes, and can also encourage persistence (Anghileri, 2006). Self-correcting tasks are also particularly useful in supporting students to make sense of a mathematical activity/concept, hence supporting their understanding (Rosen & Hoffman, 2009).

The 'art' of ratio task

The task is a hands-on paint-mixing task which is intended to identify where students are relying on additive rather than multiplicative thinking, and be sufficiently clear for students to come to a self-realisation as to whether their thinking is proportional or not.

During our trials (professional learning workshop and classroom-based trial) the task was introduced by discussing the need to mix paint to create other colours and the importance of being sure about the shade of colour you would end up with (i.e., if an artist mixes red and blue to get a particular shade of purple he or she would need to know how to re-create that exact colour again if they needed more). The class involved in the trial was a class of Year 3, 4 and 5 students (approximate ages 8–10) at a regional South Australian school. This class group consisted of 40 students in a combined Year 3/4/5 class. The class was a 60-minute mathematics lesson, of which 40 minutes of the lesson was devoted to this activity.

Participants worked in groups and each of the groups was provided with a paint mixing tray, a stirrer, and two syringes (one for each colour). Syringes were used to ensure accuracy of the volumes of paint added to each mix in order to highlight the need for precision in mathematics. The participants started by using the syringes to measure five parts yellow and one part blue into a container and mix the colours together. They then painted the colour into the square provided (see Figure 1), and documented this by giving the colour a name, indicating the ratio and saying how they worked out this ratio.

Following this they were asked to work out how much blue they would need to mix with 25 parts yellow to end up with the exact same shade of green. The students were encouraged to discuss as a group and make a decision about the mixture before they then created the colour and completed the associated record sheet. During the classroom tasks the authors and teacher moved around to the groups asking students about their created colours and the process they had undertaken. Follow-up tasks (for example, how much blue would need to be mixed with 20 parts yellow) were given on a group by group basis depending on the outcome for this first task. These included repeating this task, working with a similar task but different quantities, and working out how to create a particular volume of the exact shade of green paint they started with.

The overall response to the task, both during the teacher workshop and the classroom trial, was very positive. The teachers in the workshop considered the task to be engaging, promote understanding, and allowed them and hence their students to literally see an incorrect mix. They suggested that the task would allow for contextual and interdisciplinary learning and that there was scope within the task for a range of outcomes at different levels. The engagement level of the task was perceived (by the teachers) to be quite high, with seven of the teachers in the workshop including words like "fun", "a hands-on exploration", and "engaging" in their additional comments. They also indicated that the task had the potential to promote understanding of the concept. The visible nature of the task was also commented upon by the teachers in the workshop with comments such as "good to see the change in colours", "can ask does it look the same", and "visible outcomes". The workshop teachers did also identify some potential drawbacks to the task, in particular they were concerned with behaviour management, (for example, students squirting each other with the paint), the time it would

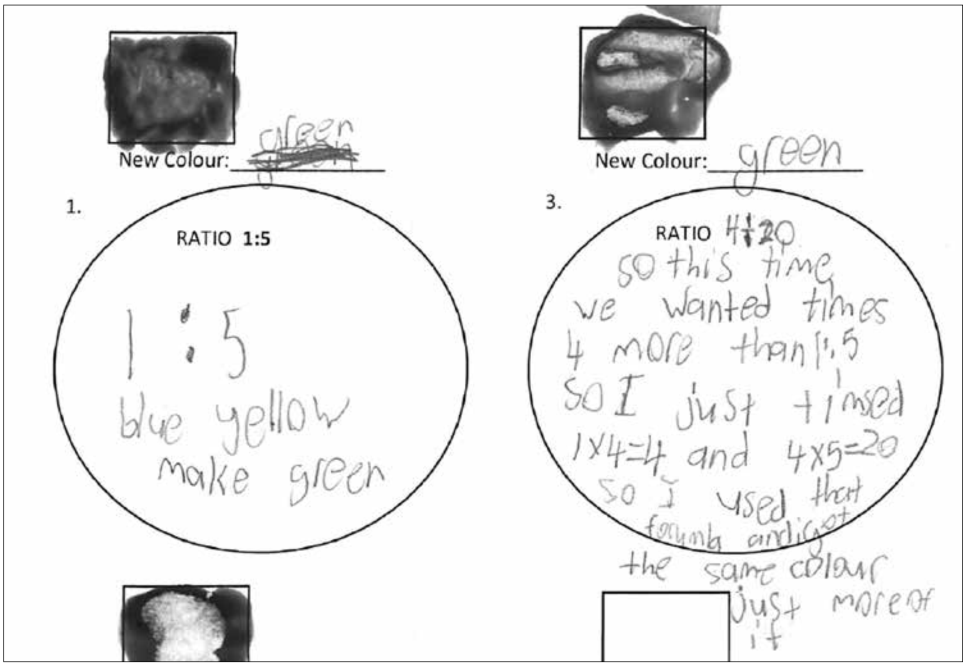

Figure 1. Example of a student response.

take to set up and clean up and the connection to the curriculum at an appropriate year level.

One of the teachers involved in the classroom trial noted that:

> … because they were actually tactile and doing the stuff their whole mindset was completely different. They were switched on, they were ready to do it and I was impressed … And it is also very real world … it is authentic to them because they know because I said to them what if I wanted to paint my house and I mixed up a sample colour using a small ratio. Now I have found the colour I want and I know what the ratio is but now I've got to make … and they were like oh yeah you would have to do something like …

Students' willingness to get involved was noted by the classroom teacher who stated:

> That was the other thing … that productive struggle … I wanted to rescue so many times. I just wanted to be like … Don't do it … [then the students would say] we did the 1:5 then the 25, then the 20 and then after that so many of them were like what's the next challenge; what's the next challenge? … I was excited because they wanted to do it …

When errors were made the students were able to recognise that the ratio was not the same, as a result of the differences in the colour. This lead to them self-correcting, and typically they were then able to amend their thinking and/or processes to get the right ratio.

> They put blues in … and they were like this doesn't look right but then they knew that they added two blues to their 25 yellow and they were like can we just add another three blues in to make it 5 to 25 and I was like yeah because that will make it right—and they did that and then it matched.

One of the authors, in their post-lesson observation, also noted that three of the groups she worked closely with initially created incorrect mixtures. One group explained that their new ratio was 21:25, demonstrating their additive thinking approach, another group had 15:25. This group had guessed '15' but then knew that it had to be too much because the colour was too dark. The third group had 18:25; they weren't really sure why they trialled 18 but explained that 5+1 was 6 and had been working on times three earlier in class so assumed it was just connected.

In the classroom trial some of the students, who initially were unsure of ratio, appeared to develop their understanding as they progressed through the activity. One example recounted by the classroom teacher was:

> I kept explaining it and explaining it and explaining it and he is like I still don't get it, I still don't get it, I still don't get it and the second we gave him the paint and he sat and watched [the student] do it then he is like oh that makes sense … it's those authentic experiences as well that like when I was saying before like these kids and you push them and you push them and then eventually that's enough and it completely changes when you offer them an activity that is authentic.

Not all of the students made sense of the colour changes, with some indicating incorrect ratios being the same. but some were able to recognise that they had the correct ratio but the colours were not the same and hence they needed to mix more thoroughly.

> I think other groups just saw it as being I'm making a paint—like they didn't get beyond that … she pointed out that oh my greens haven't come out the same and I know that it means I've mixed it wrong. So it was good that they knew they had the right measurements but they hadn't actually mixed it correctly.

Some students were able to make sense of the mixing very quickly and moved on to what the ratio would be if they required a certain quantity of paint, to paint a room for example. Two particular groups were noted in the author observations in regard to their extension tasks. They easily moved through the initial tasks, to work out the ratio of blue:yellow needed to make 90ml and 270ml of the initial shade of green.

The large class size was a potential barrier to the teacher being willing to incorporate these hands-on activities more generally, but as noted by the classroom teacher there are ways around the class size.

> Yeah absolutely so it's hard because it's such big classes here but I would—I always aim and I know [another teacher] is the same, with the math we like to be really practical. So three to four times a term we do stuff like that where

it's regular enough that the kids know it's coming but not so much that it overwhelms us as teachers.

The level of student engagement suggested by the workshop teachers and evident in the classroom trial also supports the potential of a task such as this to impact positively on student learning. As noted by the NCTM (1991) and Attard et al. (2011), deep student engagement can extend student learning and support them in developing their mathematical connections and hence impact positively on their mathematical outcomes. The teachers also identified this task as being authentic and hands-on. According to the classroom teacher, this was the core reason for the level of student engagement with the task. This task was also flexible in that it allowed for independent or collaborative work, small groups or large groups. Not surprisingly, the teachers did note some drawbacks to implementing such tasks. Of particular concern to the workshop teachers was the potential for instances of misbehaviour, however, due to the level of student engagement in the classroom trial such instances did not occur. This cannot however be attributed solely to the task as there were also two more adults present in the classroom.

All of the students in the classroom trial were able to actively engage with the activity at some level regardless of year level or prior knowledge. This echoes the responses of the teachers at the workshop, who, with the exception of the Reception, Year 1 and Year 2 class teachers, all agreed that based on their previous experience and their knowledge of their class groups the task would be appropriate for their students. This supports some of the 'readiness' literature discussed by Boyer et al. (2008) which indicated that young children can successfully negotiate proportional reasoning problems. However, it is important to note that this negotiation was completed at various levels with some students ready for more difficult extension tasks than others.

The teachers all noted the potential of this task to impact positively on student learning. One of the prominent arguments for this was the visible product, the tactile nature of the task, and that the context was something the students could relate to as being useful. This visual element of the task not only enabled the students to self-correct but also allowed for the teachers and researchers to see a representation of a student's thinking. This is not only a key feature of a good mathematical task but also supports teachers in capitalising on student learning (as encouraged by Fielding-Wells et al. (2014)) by supporting a teacher in framing the next step, question or prompt needed to scaffold and develop a student's learning. This self-correction, witnessed throughout the trial of this task, appeared to also minimise student reliance on the teacher (or researchers) to 'check-in' with regard to their answer. The students did not wait for someone to come to their group and ask them questions. Instead they would start working out what they should do to get the correct shade, allowing for more conversations about what didn't work, what they had tried and the processes they had applied, thus enhancing their engagement with the task in general.

Conclusion

This paper presents a review of a single task intended to illustrate students' understandings of ratio. The findings suggests that this task is engaging, authentic and can scaffold conceptual development of ratio. Though not a refined lesson plan or sequence of lessons, the feedback suggests that tasks such as this have the potential to enhancing student learning. The feedback from the classroom teacher also suggests that in practice this task should be done in conjunction with an art/painting class where the products could be used immediately, giving more value to the mixing activity. This would also ensure that all students were challenged to quantify their mixtures and hence enable the mathematics (in this case the concept of ratio) to have a more explicit role throughout the task, making the mathematics more obvious and more visible.

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